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PROPERTIES OF DIFFERENTIAL EQUATIONS THROUGH DIFFERENT MEASURES--ETC(U)
MAR 80 V LAKSHMIKANTHAM DAAG29-77-G-0062
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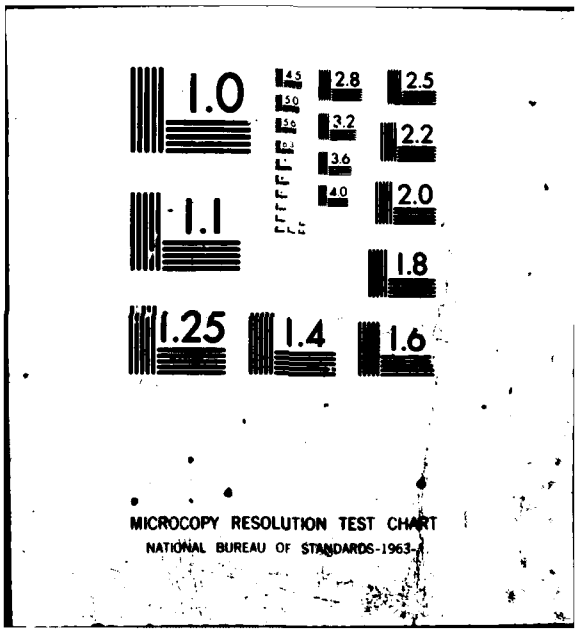
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Research is briefly summarized which addressed essentially eight areas of research: (1) reaction-diffusion equations; (2) quasi-solutions and large scale systems; (3) monotone iterative techniques; (4) Hopf bifurcation; (5) basic theory of differential equations in Banach spaces; (6) Sobolev type differential and integral equations; (7) stochastic differential equations; and (8) differential equations with infinite delay.

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## BRIEF OUTLINE OF THE MOST IMPORTANT RESULTS

There were essentially eight areas of research that we addressed ourselves to during the period January 1, 1977 to December 31, 1979. These are:

(i) reaction-diffusion equations;

(ii) quasi-solutions and large scale systems;

(iii) monotone iterative techniques;

(iv) Hopf bifurcation;

(v) basic theory of differential equations in Banach spaces;

(vi) Sobolev type differential and integral equations;

(vii) stochastic differential equations;

(viii) differential equations with infinite delay.

In the theory of differential inequalities for systems, one usually imposes a condition known as quasi-monotone property. This property is also needed in the study of large scale systems by the method of vector Lyapunov functions. However in many physical situations this property is not satisfied and consequently there has been a demand to circumvent this unpleasant difficulty. We have offered in (i) and (ii) two alternatives, namely (a) working through appropriate cones rather than $R_+^n$ and (b) using the new notion of quasi-solutions. A number of important results concerning reaction-diffusion inequalities in cones are obtained which can be utilized to prove flow-invariance, to obtain upper and lower bounds, to study the qualitative behavior such as stability and to investigate weakly coupled reaction-diffusion systems. The notion of quasi-solutions is systematically developed and the role of such solutions in the investigation of qualitative properties of large scale systems is demonstrated.

In (iii), we extended monotone iterative techniques to countable nonlinear systems of boundary value problems obtaining extremal solutions. This needed finding a generalized maximum principle. Since the method has wide applicability to systems of partial differential equations of elliptic-parabolic type and to numerical considerations via the method of lines, we generalized monotone methods to boundary value problems in Banach spaces. These results have been important and have given rise to further study by other mathematicians. There are still several open problems that are difficult and are under investigation.

The generalized Hopf bifurcation problem was considered in (iv). Results on the number of periodic orbits of differential systems in a neighborhood of a structurally unstable system (linear part has two purely imaginary eigenvalues) were given. A precise relationship between the strength of stability of the origin of the unperturbed system and the number of bifurcating orbits near the origin was provided in $R^2$. The question in $R^n$ is presently being pursued.

The item (v) deals with a number of interesting results. Since the standard cones of many infinite dimensional spaces have empty interior, contrary to known results, we have considered existence of extremal solutions of differential equations in Banach spaces, without assuming the cone has interior points. Also, we have extended Müller's results for $R^n$ to Banach spaces, that is, proving comparision theorems without assuming quasi-monotonicity. Some other important contributions deal with proving existence and uniqueness results in closed sets for differential equations with finite delay in Banach spaces. All of these results have influenced further work by other scientists.

In (vi) a new type of differential equation is studied, originally hinted by Sobolev for solving Fredholm integral equations using an imbedding technique which involves solving initial value problems for the resolvent kernel. For these Sobolev-type differential equations the following basic results were obtained: the Picard existence theorem, variation of constants formula, Gronwall type inequalities, existence of extremal solutions, a comparison principle, and Peano's type of existence result (for the Volterra integarl equations of Sobolev type). It is hoped that this new type of dynamical systems will be useful in further applications and yield interesting new properties.

In (vii) the theory of stochastic differential inequalities, minimal and maximal solutions, and a comparison principle were developed for equations of Ito type. Also considered was the connective stability analysis of Ito-type diffusion competitive-cooperative systems. This lead to results on the invariability of stability under structural perturbations, the tolerance of the complexity of the system and the reliability of the stability of the system. Equations of Ito type are basic in application to engineering and scientific problems.

The survey of equations with infinite delay given in (viii) is to provide an up-to-date account of the basic results and problems of the theory of such equations. This theory has emerged as an independent branch of the modern research in the field of mathematical analysis, with its specific problems and many connections to the applied areas. It heavily depends on the properties of function spaces, whose elements are defined on noncompact sets and on the properties of operators acting on such spaces. The study of such equations was motivated by various applications such as mechanics of continua, population

dynamics and ecology, system theory, nuclear reactor dynamics, pseudo-transport problems and neural interactions. The survey lists about 280 references and has emerged as a very important contribution.

Among other results obtained we mention a new comparison theorem for BVP's which corresponds to the well known comparison result for IVP's. Utilizing this new comparison result several results such as existence, uniqueness, convergence of successive approximations and periodic solutions have been proved for elliptic differential equations. Also new comparison theorems were obtained for studying stability and convergence properties of solutions of functional differential equations. The main techniques included Razumikhin's method as well as auxiliary functions.